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# Diagonalization of the neutralino mass matrix and boson–neutralino interaction

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**Abstract.** We analyze a connection between the neutralino mass sign, parity and structure of the neutralino–boson interaction. Correct calculation of spin-dependent and spin-independent contributions to neutralino–nuclear scattering should consider this connection. A convenient diagonalization procedure, based on the exponential parametrization of unitary matrix, is suggested.

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## 1 Introduction

Superpartners of gauge and Higgs fields play an important role in SUSY phenomenology. In particular, neutralino dark matter (DM) in the SUSY framework was considered in detail (see, for example, [1–7]), and these investigations were adapted to astrophysics. So, an analysis of the neutralino system and the structure of gauge bosons interaction with the neutralino and chargino is important for the DM description and the study of astrophysical data.

In many phenomenological works both the neutralino mass spectrum and the structure of states follow from the formal diagonalization of the neutralino mass form by an orthogonal (real) matrix [1, 8–11]. Such a procedure does not consider some important features of the structure of the Majorana states, related with the sign of the mass. These features are connected with the structure of the neutralino–boson interactions which, in turn, defines the peculiarity of the neutralino–nucleon scattering.

The most complete and comprehensive analysis of the neutralino system has been performed in [13–17]. In this work, the diagonalization of the neutralino mass matrix is considered in detail in the MSSM and some of its extensions. Special attention was paid to the building of neutralino states with positive masses. However, due to the complexity of the general diagonalization formalism it is difficult to trace a link between the sign of mass and the structure of the neutralino–nucleon interactions.

In this paper, we analyze the features of the neutralino structure and interactions which are directly related with the sign of mass. We consider the simplest case when this connection is transparent and convenient for illustration. In the second section we compare two ways of the diagonalization – by orthogonal and unitary diagonalyzing matrices. These two variants lead to neutralino states with opposite and equal signs of masses. They are formally equivalent and related by a field redefinition (see Sect. 2). But the negative mass of the neutralino (as it occurs for the first case), has to be taken into consideration in the consequent calculations. Disregarding this important feature, it is possible to get an incorrect conclusion on the spin-dependent (SD) and spin-independent (SI) contributions into the neutralino–nuclear cross section [10].

Redefinition of the field reveals a link between the sign of the mass and transformation properties of Majorana spinors with respect to inversion (i.e. parity). An analogous connection between the sign of the mass and parity was revealed for the case of massive Majorana neutrino in [18, 19]. In Sect. 3, we present the compact Lagrangian of the boson–neutralino–chargino interaction in terms of a redefined field, which is convenient for phenomenological applications.

In the fourth section, we consider the neutralino mass matrix diagonalization by means of a unitary matrix giving all positive masses. Thus, the standard calculation rules can be kept unchanged, and there is no need to check the sign of the mass or redefine the field. A convenient diagonalization procedure based on the exponential parametrization of the unitary matrix is discussed. This procedure is formalized in a perturbative calculation scheme analogous to [12]. However, our scheme needs a smaller number of input parameters and gives all expressions in a quite compact form, which is useful for calculations. The method suggested is generalized for the case of mass matrix with complex parameters (Appendix B).

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## 2 Neutralino parity and structure of boson–neutralino interaction

Now we analyze the connection between the signs of the neutralino mass and the structures of the neutralino–boson interaction when $M_Z/M_k \to 0$ and $M_k$ is $M_1, M_2$ or $\mu$. In this limit, the analysis is simplified considerably, but the results can be used in the general case too. This limit is approximately realized in split SUSY scenarios [3] and strictly takes place at high temperatures $T \gg E_{\text{EW}}$, when the Higgs condensate is melted (the high symmetry phase). For completeness, we give the well-known minimal formalism that we need in the following analysis.

If the mixing of gauge and Higgs fermions is neglected, the mass term of higgsino-like Majorana fields has the Dirac form [20]:

$$M_h = \frac{1}{2}\mu\left(\bar{H}^0_{1R}H^0_{2\text{L}} + \bar{H}^0_{2\text{R}}H^0_{1\text{L}}\right) + \text{h.c.} \tag{1}$$

This form can be represented by a $(2\times 2)$-mass matrix which is known as the specific matrix with zero trace:

$$\mathbf{M}_2 = \begin{pmatrix} 0 & \mu \\ \mu & 0 \end{pmatrix}. \tag{2}$$

There are two ways to diagonalize this matrix. The formal procedure using the orthogonal matrix $\mathbf{O}_2$ leads to a spectrum with opposite signs:

$$\mathbf{O}_2^{\text{T}}\mathbf{M}_2\mathbf{O}_2 = \begin{pmatrix} \mu & 0 \\ 0 & -\mu \end{pmatrix}, \quad \mathbf{O}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix},$$
$$m_a = (\mu, -\mu)\,, \tag{3}$$

where $\text{Tr}\{\mathbf{O}_2^{\text{T}}\mathbf{M}_2\mathbf{O}_2\} = \text{Tr}\{\mathbf{M}_2\} = 0$ (trace conservation). In this case, one of the Majorana fields has a negative mass, regardless of the sign of $\mu$. The matrix $\mathbf{M}_2$ can also be diagonalized by the unitary complex matrix $\mathbf{U}_2$, giving masses with the same sign:

$$\mathbf{U}_2^{\text{T}}\mathbf{M}_2\mathbf{U}_2 = \begin{pmatrix} \mu & 0 \\ 0 & \mu \end{pmatrix}, \quad \mathbf{U}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \text{i} \\ 1 & -\text{i} \end{pmatrix},$$
$$m_a = (\mu, \mu)\,. \tag{4}$$

The diagonalization (4) is equivalent to the procedure (3) with the redefinition $\chi \to \text{i}\gamma_5\chi$ of the non-chiral (full) field with $m = -\mu$. The last transformation is equivalent to $\chi_{\text{R,L}} \to \pm\text{i}\chi_{\text{R,L}}$ for the chiral components. Note that there is an infinite set of unitary matrices $\mathbf{U}_\phi = \mathbf{U}_2 \cdot \mathbf{O}_\phi$ which diagonalize the mass matrix $\mathbf{M}_2$ (see also [15], Appendix A.2):

$$\mathbf{U}_\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \text{e}^{\text{i}\phi} & \text{i}\text{e}^{\text{i}\phi} \\ \text{e}^{-\text{i}\phi} & -\text{i}\text{e}^{-\text{i}\phi} \end{pmatrix}, \quad \mathbf{O}_\phi = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}. \tag{5}$$

The additional $O_2$-symmetry (see Appendix B) leads to a free parameter arising in the general case.

Dealing with the spinor field we should take into account the sign of its mass in the propagator and polarization matrix or redefine the field with a negative mass. As a rule this feature is not considered in phenomenological applications (see, for example, [8–11]). From the redefinition $\chi' = \text{i}\gamma_5\chi$, it follows that the transformation (relative to inversion) properties of Majorana fields having opposite mass signs are different. As a result, we have one usual Majorana field and one pseudo-Majorana field.

The gaugino mass subform is of the standard Majorana type [20] and has no specific features. The signs of the masses for $\chi_1$ and $\chi_2$ are defined by the signs of $M_1$ and $M_2$ in the case of small mixing. They can be made positive by a redefinition. Note that the redefinition procedure always influences the mixing terms of the mass matrix and should be taken into account in the general case (see Sect. 4).

Now we consider the connection between the structure of the boson–neutralino interaction and the relative sign of the neutralino masses. For simplicity, we show this connection in the pure higgsino approximation. The contribution of terms caused by mixing is considered in the next section. We here present a short comparative analysis of the calculation rules in two cases: when the masses of $\chi_3$ and $\chi_4$ have different signs (diagonalization (3)) and when they have the same signs (diagonalization (4)). The initial Lagrangian is

$$L_{\text{int}} = \frac{1}{2}g_Z Z_\mu\left(\bar{H}^0_{1\text{L}}\gamma^\mu H^0_{1\text{L}} + \bar{H}^0_{2\text{R}}\gamma^\mu H^0_{2\text{R}}\right), \tag{6}$$

where $g_Z = g_2/\cos\theta_{\text{W}}$. The diagonalizations (3) and (4) lead to the following forms of neutralino–boson interactions, respectively:

$$(1) \quad L_{\text{int}} = -\frac{1}{2}g_Z Z_\mu \bar{\chi}_3\gamma^\mu\gamma_5\chi'_4\,;$$
$$(2) \quad L_{\text{int}} = \frac{\text{i}}{2}g_Z Z_\mu \bar{\chi}_3\gamma^\mu\chi_4\,. \tag{7}$$

In (7) the first case with opposite signs $(\mu, -\mu)$ can be transformed into the second case with the same signs $(\mu, \mu)$ by the redefinition $\text{i}\gamma_5\chi'_4 = \chi_4$. Here we show that both Lagrangians in (7) lead to the same result without any field redefinition if the negative sign of $\chi'_4$ mass is considered in the calculations. In other words, both structures in (7) lead to the parity-conserving vector interaction which gives the spin-independent contribution to the neutralino–nucleon scattering [21].

Let us consider, for example, the process of the scattering $\chi_3 q \to \chi_4 q$ with $t$-channel exchange of a $Z$-boson ($t^2 \ll M_Z^2$). In both cases, the amplitudes of this process are

$$(1) \quad \mathcal{M}_1 \sim \bar{\chi}'^{+}_4(p_2)\gamma^\mu\gamma_5\chi^-_3(p_1)\cdot\bar{q}^+(k_2)\gamma_\mu(c_q - \gamma_5)q^-(k_1)\,,$$
$$(2) \quad \mathcal{M}_2 \sim \bar{\chi}^{+}_4(p_2)\gamma^\mu\chi^-_3(p_1)\cdot\bar{q}^+(k_2)\gamma_\mu(c_q - \gamma_5)q^-(k_1)\,. \tag{8}$$

Formally, the amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ have a different structure. Therfore, one can draw a wrong conclusion about the contributions to the spin-dependent and spin-independent parts of the cross section, if the negative sign of $\chi'_4$ mass has not been taken into account. However, taking into account the negative sign of $\chi'_4$ in the polarization matrix allows one to get the same result for both cases. If $\chi$ is in an initial or final state, the polarization matrix of the field $\chi$ in $\mathcal{M}^+\mathcal{M}$ is defined by (for positive mass $m_\chi = \mu > 0$)

$$\sum_\sigma \chi_\sigma^\mp(p)\bar{\chi}_\sigma^\pm(p) = \frac{1}{2p^0}(\hat{p} \pm \mu)\,, \tag{9}$$

or (for negative mass $m_{\chi'} = -\mu$)

$$\sum_\sigma \chi_\sigma'^\mp(p)\bar{\chi}_\sigma'^\pm(p) = \frac{1}{2p^0}(\hat{p} \mp \mu)\,. \tag{10}$$

With the help of (10), we get

$$\begin{aligned}\mathcal{M}_1^+\mathcal{M}_1 &\sim \mathrm{Tr}\left\{(\hat{p}_2 - \mu)\gamma^\mu\gamma_5(\hat{p}_1+\mu)\gamma^\nu\gamma_5\right\}\\ &= \mathrm{Tr}\left\{(\hat{p}_2 + \mu)\gamma^\mu(\hat{p}_1+\mu)\gamma^\nu\right\}.\end{aligned} \tag{11}$$

One can get the same expression for $\mathcal{M}_2^+\mathcal{M}_2$ using the standard definition (9) of the polarization matrix. This feature should be included in an analysis of neutralino–nucleon scattering. From the interaction Lagrangian only, without consideration of the mass signs, we cannot draw any valuable conclusions on the SD or SI contributions. In particular, the bilinear structures $\bar{\chi}_3\gamma_\mu\chi_4$ and $\bar{\chi}_3\gamma^\mu\gamma_5\chi_4'$ are vectors, while $\bar{\chi}_3\gamma_\mu\chi_4'$ and $\bar{\chi}_3\gamma^\mu\gamma_5\chi_4$ are axial vectors. Analogously, $\bar{\chi}_3\chi_4'$ and $\bar{\chi}_3\gamma_5\chi_4$ are pseudoscalars, while $\bar{\chi}_3\gamma_5\chi_4'$ and $\bar{\chi}_3\chi_4$ are scalars. Thus, the analysis of the neutralino–nucleon interaction has to take into account neutralino transformation properties. As a rule, in the bulk of papers this feature has not been considered explicitly and mistaken conclusions can be obtained in calculations of the SD and SI contribution to the neutralino–nucleon interaction. In particular, for the current structure $\bar{\chi}_i\gamma^\mu\gamma_5\chi_k Z_\mu$ it is possible to obtain SD or SI neutralino–nucleon cross sections depending on the neutralino relative parity. For instance, in [10, 22, 23] the same current structure was considered without any comments on this important peculiarity. From our analysis, it follows that in the case discussed, the neutralino–boson interaction gives the main contribution to the spin-independent part of the cross section [21].

An analogous feature is in order when $\chi_4'$ is in an intermediate state, for example, in the process $\chi_3 Z \to \chi_4' \to \chi_3 Z$. The amplitude of the process is

$$\begin{aligned}\mathcal{M}_1 &\sim \bar{\chi}_3^+(p_2)\gamma^\mu\gamma_5(\hat{q}-\mu)\gamma^\nu\gamma_5\chi_3^-(p_1)e_\mu^Z e_\nu^Z\\ &= \bar{\chi}_3^+(p_2)\gamma^\mu(\hat{q}+m_\chi)\gamma^\nu\chi_3^-(p_1)e_\mu^Z e_\nu^Z\,.\end{aligned} \tag{12}$$

In (12) we use the propagator $\sim(\hat{q}-\mu)$ for the field $\chi_4'$ with negative mass $m_{\chi'} = -\mu$, whereas the standard propagator is $\sim(\hat{q}+\mu)$. So, the mass sign being taken in account leads to the same result for the amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$, where $\mathcal{M}_2$ describes the same process with redefined $\chi_4$ in an intermediate state.

## 3 Gauge boson–neutralino–chargino interactions

In this section, we give compact expressions for the Lagrangian of gauge boson–neutralino–chargino interactions in the case of small mixing. These expressions are convenient for calculations in cosmology. This Lagrangian follows from (A.14)–(A.16) as a result of the shift

$$\begin{aligned}L_{\mathrm{int}} = &\frac{\mathrm{i}}{2}g_2\epsilon_{abc}\bar{W}^a\gamma^\mu W^c W_\mu^b - \frac{1}{2}g_1\bar{H}_1^-\gamma^\mu H_{1\mathrm{L}}^- B_\mu\\ &+\frac{1}{2}g_1\bar{H}_2^+\gamma^\mu H_{2\mathrm{L}}^+ B_\mu\\ &+\frac{1}{\sqrt{2}}g_2 W_\mu^+\left(\bar{H}_1^0\gamma^\mu H_{1\mathrm{L}}^- + \bar{H}_2^+\gamma^\mu H_{2\mathrm{L}}^0\right)\\ &+\frac{1}{\sqrt{2}}g_2 W_\mu^-\left(\bar{H}_1^-\gamma_\mu H_{1\mathrm{L}}^0 + \bar{H}_2^0\gamma^\mu H_{2\mathrm{L}}^+\right)\\ &+\frac{1}{2}g_2 W_\mu^3\big(-\bar{H}_1^-\gamma^\mu H_{1\mathrm{L}}^- + \bar{H}_2^+\gamma^\mu H_{2\mathrm{L}}^+\\ &+\bar{H}_1^0\gamma^\mu H_{1\mathrm{L}}^0 - \bar{H}_2^0\gamma^\mu H_{2\mathrm{L}}^0\big)\\ &-\frac{1}{2}g_1\bar{H}_1^0\gamma^\mu H_{1\mathrm{L}}^0 B_\mu + \frac{1}{2}g_1\bar{H}_2^0\gamma^\mu H_{2\mathrm{L}}^0 B_\mu\,.\end{aligned} \tag{13}$$

Let us consider the case $M_Z \ll \mu, M_{1,2}$, which can be used in split SUSY models [2–4, 10]. The physical states of the neutralino in the zeroth order of the mixing were defined in Sect. 2, and the chargino states in Appendix B:

$$\begin{aligned}&\chi_1 = W^3\,,\quad \chi_2 = B\,,\quad \chi_3 = \left(H_1^0 + H_2^0\right)/\sqrt{2}\,,\\ &\chi_4 = \mathrm{i}\gamma_5\left(H_1^0 - H_2^0\right)/\sqrt{2}\,;\\ &\tilde{H} = -\mathrm{i}\gamma_5(H_{1\mathrm{L}}^- + H_{2\mathrm{R}}^+)\,,\quad \tilde{W} = (W_1 + \mathrm{i}W_2)/\sqrt{2}\,.\end{aligned} \tag{14}$$

In (14) we do not use charge sign notation for the Dirac fields $\tilde{H}$ and $\tilde{W}$ (in contrast to $W_\mu^\pm$) in analogy to the standard model notation. From the structure of $\tilde{H}$ in (14), it follows that the components $H_{1\mathrm{L}}^-$ and $H_{2\mathrm{R}}^+$ correspond to particle and anti-particle parts in a Weyl basis. Using the definitions (14) we represent $L_{\mathrm{int}}$ in the form

$$\begin{aligned}L_{\mathrm{int}} = &g_2 W_\mu^+\left(\bar{\chi}_1\gamma^\mu\tilde{W} - \frac{\mathrm{i}}{2}\bar{\chi}_3\gamma^\mu\tilde{H} - \frac{1}{2}\bar{\chi}_4\gamma^\mu\tilde{H}\right)\\ &+g_2 W_\mu^-\left(\bar{\tilde{W}}\gamma^\mu\chi_1 + \frac{\mathrm{i}}{2}\bar{\tilde{H}}\gamma^\mu\chi_3 - \frac{1}{2}\bar{\tilde{H}}\gamma^\mu\chi_4\right)\\ &-g_2\cos\theta_{\mathrm{W}} Z_\mu\bar{\tilde{W}}\gamma^\mu\tilde{W} - \frac{g_2}{2\cos\theta_{\mathrm{W}}}\cos 2\theta_{\mathrm{W}} Z_\mu\bar{\tilde{H}}\gamma^\mu\tilde{H}\\ &+\frac{\mathrm{i}g_2}{2\cos\theta_{\mathrm{W}}}Z_\mu\bar{\chi}_3\gamma^\mu\chi_4 - eA_\mu\bar{\tilde{W}}\gamma^\mu\tilde{W} - eA_\mu\bar{\tilde{H}}\gamma^\mu\tilde{H}\,.\end{aligned} \tag{15}$$

The first order corrections to the $Z\chi_i\chi_k$ interaction caused by the mixing (see Appendix B) are

$$\begin{aligned}L_{\mathrm{mix}}^{(1)} = &\frac{g_2}{2\cos\theta_{\mathrm{W}}}Z_\mu\left(-\frac{\mathrm{i}m_2}{M_1-\mu}\bar{\chi}_1\gamma^\mu\chi_3 + \frac{\mathrm{i}m_4}{M_2-\mu}\bar{\chi}_2\gamma^\mu\chi_3\right.\\ &\left.-\frac{m_1}{M_1+\mu}\bar{\chi}_1\gamma^\mu\gamma_5\chi_4 + \frac{m_3}{M_2+\mu}\bar{\chi}_2\gamma^\mu\gamma_5\chi_4\right)\,,\end{aligned} \tag{16}$$

where the $m_k$ are defined by (22) in the next section. From (16), one can see that the interactions of $\chi_3$ and $\chi_4$ with $\chi_{1,2}$ have a different structure. This effect is directly connected with different signs of the masses of the non-redefined fields. Note also that the bino-like neutralino $\chi_2 \approx B$ does not interact with gauge bosons in the zero mixing approximation, but it interacts with the scalar Higgs field and $\chi_{3,4}$.

In the pure higgsino limit, $\chi_3$ and $\chi_4$ constitute the neutral Dirac field $\tilde{H}^0 = (\chi_3 + \mathrm{i}\chi_4)/\sqrt{2}$ and the part of (15) can be represented in the form (here we omit the heavy states $\tilde{W}$ and $\chi_2$)

$$\begin{aligned} L^{\mathrm{D}}_{\mathrm{int}} = &-\frac{\mathrm{i}g_2}{2} W^+_\mu \bar{\tilde{H}}^0 \gamma^\mu \tilde{H} + \frac{\mathrm{i}g_2}{2} W^-_\mu \bar{\tilde{H}} \gamma^\mu \tilde{H}^0 \\ &+ \frac{g_2}{2\cos\theta_{\mathrm{W}}} Z_\mu \bar{\tilde{H}}^0 \gamma_\mu \tilde{H}^0 \\ &- \frac{g_2}{2\cos\theta_{\mathrm{W}}} \cos 2\theta_{\mathrm{W}} Z_\mu \bar{\tilde{H}} \gamma_\mu \tilde{H} - eA_\mu \bar{\tilde{H}} \gamma^\mu \tilde{H}\,. \end{aligned} \tag{17}$$

The Dirac representation (17) of the boson–neutralino–chargino interactions involving a small mixing of the Higgs fermion with the gauge ones is formal (unphysical) but is convenient for our calculations. In this case, we avoid some complications of the Feynman rules, caused by the Majorana nature of $\chi_3$ and $\chi_4$ [24–26]. By direct calculation we have checked that both ways lead to the same results for the annihilation and co-annihilation cross sections [27].

## 4 Diagonalization of the neutralino mass matrix by unitary matrix with exponential parametrization

In this section, we consider diagonalization of the $4\times 4$ mass matrix with real parameters $\mu, M_1, M_2$. Generalization of the approach for a matrix with complex parameters is considered in Appendix B. As follows from Sect. 2, the sign $\mu$ is not essential, and $M_{1,2}$ can be made positive by a redefinition of the gauge fermion. The neutral fermion mass form follows from the SUSY Lagrangian ((A.15) and (A.16)) after the shift

$$L_m = -\frac{1}{2}(\bar{\phi}_{\mathrm{R}})^{\mathrm{T}} \mathbf{M}_0 \phi_{\mathrm{L}} + \mathrm{h.c.}\,, \tag{18}$$

where $(\phi)^{\mathrm{T}} = (B, W^3, H^0_1, H^0_2)$ and

$$\mathbf{M}_0 = \begin{pmatrix} M_1 & 0 & -\mathrm{i}M_Z s_\theta c_\beta & \mathrm{i}M_Z s_\theta s_\beta \\ 0 & M_2 & \mathrm{i}M_Z c_\theta c_\beta & -\mathrm{i}M_Z c_\theta s_\beta \\ -\mathrm{i}M_Z s_\theta c_\beta & \mathrm{i}M_Z c_\theta c_\beta & 0 & \mu \\ \mathrm{i}M_Z s_\theta s_\beta & -\mathrm{i}M_Z c_\theta s_\beta & \mu & 0 \end{pmatrix}, \tag{19}$$

where $s_\theta = \sin\theta$ and $c_\beta = \cos\beta$. The matrix (19) differs from the commonly used one by the presence of the imaginary unit in the mixing terms. One can go to a real traditional matrix $M'_0$ by a redefinition $H'_a = \mathrm{i}\gamma_5 H_a$, where $a = 1, 2$. As a result we get the standard matrix following from (19) under the formal transition $\mathrm{i}M_Z \to M_Z$ and $\mu \to -\mu$. However, implying our diagonalization procedure there is no need to do this transformation [23].

It is convenient to analyze the diagonalization of the matrix (19) with the help of the intermediate transformation

$$\mathbf{M}_{\mathrm{I}} = \mathbf{U}^{\mathrm{T}}_{\mathrm{I}} \mathbf{M}_0 \mathbf{U}_{\mathrm{I}}\,; \quad \mathbf{U}_{\mathrm{I}} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_2 \end{pmatrix}. \tag{20}$$

Here $\mathbf{1}$ and $\mathbf{0}$ are the identity and zero $(2\times 2)$-matrices, and $\mathbf{U}_2$ is defined by (4) in the pure higgsino limit. Then the intermediate mass matrix has the form

$$\mathbf{M}_{\mathrm{I}} = \begin{pmatrix} M_1 & 0 & -\mathrm{i}m_1 & m_2 \\ 0 & M_2 & \mathrm{i}m_3 & -m_4 \\ -\mathrm{i}m_1 & \mathrm{i}m_3 & \mu & 0 \\ m_2 & -m_4 & 0 & \mu \end{pmatrix}, \tag{21}$$

where

$$\begin{aligned} m_1 &= M_Z \sin\theta_{\mathrm{W}} (\cos\beta - \sin\beta)/\sqrt{2}\,, \\ m_2 &= M_Z \sin\theta_{\mathrm{W}} (\cos\beta + \sin\beta)/\sqrt{2}\,, \\ m_3 &= M_Z \cos\theta_{\mathrm{W}} (\cos\beta - \sin\beta)/\sqrt{2}\,, \\ m_4 &= M_Z \cos\theta_{\mathrm{W}} (\cos\beta + \sin\beta)/\sqrt{2}\,. \end{aligned} \tag{22}$$

Intermediate fields are defined by $\phi_{\mathrm{I}} = \mathbf{U}_{\mathrm{I}}\phi$; that is, $(\phi_{\mathrm{I}})^{\mathrm{T}} = (B, W^3, \chi^{\mathrm{I}}_3, \chi^{\mathrm{I}}_4)$, where $\chi^{\mathrm{I}}_3$ and $\chi^{\mathrm{I}}_4$ are defined by (7). The use of the intermediate mass matrix provides the positivity of the higgsino-like neutralino masses and leads to the "quasidiagonal" structure of the matrix in the case of small mixing.

The matrix (21) is symmetric and complex, but it is not Hermitian. The spectrum of $\mathbf{M}_{\mathrm{I}}$ is real and has a simple form. However, it is not a mass spectrum of the neutralino, because the diagonalization of the neutralino mass matrix $\mathbf{U}^{\mathrm{T}}_{\mathrm{I}} \mathbf{M}_{\mathrm{I}} \mathbf{U}_{\mathrm{I}} = \mathrm{diag}(m_k)$ differs from the one defined by $\mathbf{U}^+ \mathbf{M}_{\mathrm{I}} \mathbf{U} = \mathrm{diag}(\lambda_k)$. In the last case, $\mathbf{U}$ is built of the eigenvectors of $\mathbf{M}_{\mathrm{I}}$ and the $\lambda_k$ are the eigenvalues of $\mathbf{M}_{\mathrm{I}}$. According to Theorem 4.4.4 (Takagi expansion) from [28], any complex symmetric matrix can be diagonalized by the unitary matrix $\mathbf{U}$:

$$\mathbf{U}^+ \mathbf{M} \mathbf{U}^* = \mathrm{diag}(m_k)\,, \quad m_k > 0\,, \tag{23}$$

where $\mathbf{U}$ is built from eigenvectors of the matrix $\mathbf{A} = \mathbf{M}\mathbf{M}^*$ with the spectrum $\{m^2_k\}$, i.e. $\mathbf{U}^+ \mathbf{A} \mathbf{U} = \mathrm{diag}(m^2_k)$. Consistency of the last relation and (23) is evident from the equality

$$\mathbf{U}^+ \mathbf{M} \mathbf{U}^* (\mathbf{U}^*)^{-1} \mathbf{M}^* \mathbf{U} = \mathrm{diag}(m^2_k)\,, \quad m^*_k = m_k\,. \tag{24}$$

The method based on the Takagi theorem was considered in [12, 15, 29, 30], where the standard way of the determination of the spectrum is given. However, there is no need to solve this complicated problem in the case considered. Here we show that the spectrum of matrix $\mathbf{A} = \mathbf{M}_{\mathrm{I}} \mathbf{M}^+_{\mathrm{I}}$ coincides with the squared spectrum of the traditional real mass matrix $\mathbf{M}_0'$. The spectrum of the matrix $\mathbf{A}=\mathbf{M}_\mathrm{I}\mathbf{M}_\mathrm{I}^+$ ($\mathbf{M}_\mathrm{I}$ is defined by (21)) follows from the solution of the characteristic equation $\det(\mathbf{A}-\lambda\cdot\mathbf{1})=0$,

$$\lambda^4-a\lambda^3+b\lambda^2-c\lambda+d=0\,. \qquad (25)$$

The coefficients $a,b,c$, and $d$ in (25) are expressed in terms of the matrix elements of $\mathbf{M}_0$ as follows:

$$\begin{aligned}
a&=M_1^2+M_2^2+2\mu^2+2M_Z^2;\\
b&=M_1^2M_2^2+2M_1^2\left(\mu^2+M_Z^2\cos^2\theta_\mathrm{W}\right)\\
&\quad+2M_2^2\left(\mu^2+M_Z^2\sin^2\theta_\mathrm{W}\right)\\
&\quad+2M_Z^2\mu\sin 2\beta\left(M_1\sin^2\theta_\mathrm{W}+M_2\cos^2\theta_\mathrm{W}\right)\\
&\quad+\left(\mu^2+M_Z^2\right)^2;\\
c&=2\mu^2M_1^2M_2^2+M_1^2\left[\left(\mu^2+M_Z^2\cos^2\theta_\mathrm{W}\right)^2\right.\\
&\quad\left.+2M_2M_Z^2\mu\cos^2\theta_\mathrm{W}\sin 2\beta\right]\\
&\quad+M_2^2\left[\left(\mu^2+M_Z^2\sin^2\theta_\mathrm{W}\right)^2+2M_1M_Z^2\mu\sin^2\theta_\mathrm{W}\sin 2\beta\right]\\
&\quad+\frac{1}{2}M_1M_2M_Z^4\sin^2 2\theta_\mathrm{W}\\
&\quad+2M_Z^2\mu^3\sin 2\beta\left(M_1\sin^2\theta_\mathrm{W}+M_2\cos^2\theta_\mathrm{W}\right)\\
&\quad+\mu^2M_Z^4\sin^2 2\beta;\\
d&=\mu^4M_1^2M_2^2+M_1^2\mu^2M_Z^2\left(2\mu M_2+M_Z^2\cos^2\theta_\mathrm{W}\sin 2\beta\right)\\
&\quad\times\cos^2\theta_\mathrm{W}\sin 2\beta\\
&\quad+M_2^2\mu^2M_Z^2\left(2\mu M_1+M_Z^2\sin^2\theta_\mathrm{W}\sin 2\beta\right)\sin^2\theta_\mathrm{W}\sin 2\beta\\
&\quad+\frac{1}{2}M_1M_2\mu^2M_Z^4\sin^2 2\theta_\mathrm{W}\sin^2 2\beta\,.
\end{aligned} \qquad (26)$$

Analogous expressions are given in [12], where the algorithm of the definition of the spectrum $\lambda_k$ is considered. In the general case, we can get exact expressions for the roots $\lambda_k$ of (25) in terms of its general algebraic solutions. It is difficult to analyze and compare such expressions, but we can show that the roots of (25) are $\lambda_k=m_k^2$, where $m_k$ is the conventional neutralino spectrum. To show this, let us write the characteristic equation, $\det(\mathbf{M}_0'-l\cdot\mathbf{1})=0$, in the form ($\mathbf{M}_0'$ is the standard real mass matrix [30, 32, 33])

$$\begin{aligned}
&(M_1-l)(M_2-l)(l^2-\mu^2)\\
&\quad+M_Z^2(l+\mu\sin 2\beta)\left(M_1\cos^2\theta_\mathrm{W}+M_2\sin^2\theta_\mathrm{W}-l\right)=0\,.
\end{aligned} \qquad (27)$$

Then we arrange the even and odd degrees of $l$ on the left and right hand sides of this equation separately. Squaring the equation and changing $l_k^2\to\lambda_k$, we get (25) with coefficients (26). Moreover, by direct calculation we have checked that the mass spectrum appearing as a result of the diagonalization

$$\mathbf{U}^+\mathbf{M}_\mathrm{I}\mathbf{U}^*=\mathrm{diag}(m_k) \qquad (28)$$

is entirely positive (see Appendix B).

With the help of the Takagi theorem it is possible to illustrate the correct construction of the positive mass spectrum. However, the above discussed method is not convenient for calculations and can be used as the diagonalizability proof only: there is a unitary matrix with the property (23) or (28). The use of $\mathbf{M}_\mathrm{I}$ gives a convenient tool for the calculation of the spectrum and states when $M_1, M_2, \mu$ and their differences are much greater than $M_Z$. The hierarchy of $M_1$, $M_2$ and $\mu$ is arbitrary, i.e. one can apply the method suggested to the various scenarios of the neutralino DM. In this case, the diagonalyzing matrix is quasidiagonal, i.e. $|U_{kk}|\approx 1$ and $|U_{ik}|\ll 1$, $i\neq k$. Then we represent this matrix in the exponential form which contains six angles and six phases as input parameters. A similar approach was considered in the general case in [12], where six angle and ten phase parameters were applied. Here we show that in the case of the mass matrix (21) it is possible to use six phases only (see Appendix B):

$$\mathbf{U}=\begin{pmatrix}
a_1 & \delta_1\mathrm{e}^{-\mathrm{i}\phi_1} & \delta_2\mathrm{e}^{-\mathrm{i}\phi_2} & \delta_3\mathrm{e}^{-\mathrm{i}\phi_3}\\
r_1\mathrm{e}^{\mathrm{i}\alpha_1} & a_2 & \delta_4\mathrm{e}^{-\mathrm{i}\phi_4} & \delta_5\mathrm{e}^{-\mathrm{i}\phi_5}\\
r_2\mathrm{e}^{\mathrm{i}\alpha_2} & r_4\mathrm{e}^{\mathrm{i}\alpha_4} & a_3 & \delta_6\mathrm{e}^{-\mathrm{i}\phi_6}\\
r_3\mathrm{e}^{\mathrm{i}\alpha_3} & r_5\mathrm{e}^{\mathrm{i}\alpha_5} & r_6\mathrm{e}^{\mathrm{i}\alpha_6} & a_4
\end{pmatrix}, \qquad (29)$$

where $\delta_k$ and $\phi_k$ are input angle and phase parameters. The values $a_\beta, r_i$ and $\alpha_k$ are some functions of the input parameters which are defined by the unitary conditions $(\mathbf{U}^+\mathbf{U})_{ik}=\delta_{ik}$. In our case, $|\delta_k|\ll 1$ and functions $a_\beta, r_i$ and $\alpha_k$ are easily determined by successive approximations (Appendix B). Apparently, the diagonalization of the real matrix demands the angle parameters only. Having used the diagonalization conditions

$$\mathbf{U}_\mathrm{I}^\mathrm{T}\mathbf{M}_\mathrm{I}\mathbf{U}_\mathrm{I}=\mathbf{M}_\mathrm{d}\equiv\mathrm{diag}(m_1,m_2,m_3,m_4) \qquad (30)$$

the input parameters $\delta_i$ and $\phi_k$ can be determined from the six independent equations $(\mathbf{M}_\mathrm{d})_{ik}=0$, $i>k$. So, there are six conditions for the real and six ones for the imaginary parts of matrix elements. Then the masses $m_\alpha$ appear as functions of the defined input parameters. As is shown in Appendix B, the perturbative calculation scheme can easily be formalized.

The above discussed method of diagonalization is applied to the case of a mass matrix with complex parameters $M_1\mathrm{e}^{\mathrm{i}\psi_1}$, $M_2\mathrm{e}^{\mathrm{i}\psi_2}$ and $\mu\mathrm{e}^{\mathrm{i}\psi_\mu}$ (see Appendix B). In this case, we have to generalize (29) introducing additional phase parameters according to $a_k\to a_k\mathrm{e}^{\mathrm{i}\xi_k}$. Thus, we have the same quantity of parameters as in [12]. The functions $a_\beta, r_k$ and $\alpha_k$ are determined in terms of the input parameters $\delta_k, \phi_k$ and $\xi_\beta$ using the unitary condition $\mathbf{U}^+\mathbf{U}=\mathbf{1}$. The input parameters are determined in terms of the mass matrix elements if we use the diagonalization conditions $(\mathbf{U}^\mathrm{T}\mathbf{M}\mathbf{U})_{ik}=0$, $i\neq k$ and $\mathrm{Im}(\mathbf{U}^\mathrm{T}\mathbf{M}\mathbf{U})_{ii}=0$. Note that in this case the perturbative calculation scheme (as for the real mass matrix) is in order also. However, the expressions are more complicated and bulky, so we give the results in the first approximation only (Appendix B).

Here we represent the mass spectrum and parameters of the matrix $\mathbf{U}$ defined by (29), up to terms $\sim m_Z^2/M_a^2$, $a=1,2$ (Appendix B). The neutralino masses are

$$\begin{aligned}
m_{\chi_1}&=M_1+\frac{M_Z^2\sin^2\theta_\mathrm{W}}{M_1^2-\mu^2}(M_1+\mu\sin 2\beta)\,,\\
m_{\chi_2}&=M_2+\frac{M_Z^2\cos^2\theta_\mathrm{W}}{M_2^2-\mu^2}(M_2+\mu\sin 2\beta)\,,
\end{aligned}$$

$$m_{\chi_3} = \mu + \frac{M_Z^2(1-\sin 2\beta)}{2(M_1+\mu)(M_2+\mu)} \times \left(M_1 \cos^2\theta_{\rm W} + M_2 \sin^2\theta_{\rm W} + \mu\right),$$
$$m_{\chi_4} = \mu - \frac{M_Z^2(1+\sin 2\beta)}{2(M_1-\mu)(M_2-\mu)} \times \left(M_1 \cos^2\theta_{\rm W} + M_2 \sin^2\theta_{\rm W} - \mu\right). \quad (31)$$

From (31) one can see that $m_{\chi_3}$ and $m_{\chi_4}$ have the same sign. The validity of the expressions (31) does not depend on the hierarchy of $M_a$ and $\mu$. So one can use (31) in various SUSY scenarios.

Another feature of the diagonalization is the presence of a free parameter in the structure of neutralino states (Appendix B). Evidently, this free parameter is a remainder of $O_2$-symmetry in the pure higgsino limit, and it does not enter into expressions for the masses (the last assertion is checked by direct calculation in the second order approximation).

The structure of the neutralino chiral fields follows from the transformations

$$\phi_{\rm L} = \mathbf{U}\chi_{\rm L}, \qquad \phi_{\rm R} = (\phi_{\rm L})^{\rm C} = \mathbf{U}^*\chi_{\rm R},$$
$$\chi_{\rm L} = \mathbf{U}^{-1}\phi_{\rm L} = \mathbf{U}^+\phi_{\rm L}, \quad \chi_{\rm R} = (\chi_{\rm L})^{\rm C} = \mathbf{U}^{\rm T}\phi_{\rm R}, \quad (32)$$

where $\mathbf{U} = \mathbf{U}_2 \cdot \mathbf{U}_{\rm I}$ and $(\phi)^{\rm T} = (B, W^3, H_1^0, H_2^0)$. With the help of (32) for the non-chiral neutralino field $\chi = \chi_{\rm L} + \chi_{\rm R}$, we get

$$\phi = (\operatorname{Re}\mathbf{U} - \mathrm{i}\gamma_5 \operatorname{Im}\mathbf{U})\chi, \quad \chi = \left(\operatorname{Re}\mathbf{U}^{\rm T} + \mathrm{i}\gamma_5 \operatorname{Im}\mathbf{U}^{\rm T}\right)\phi. \quad (33)$$

In the first order of mixing (see Appendix B) the structure of the neutralino fields is defined by the following expressions

$$\chi_1 \approx B + \frac{\mathrm{i}}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} + \frac{m_2}{M_1-\mu}\right)\gamma_5 H_1^0 + \frac{\mathrm{i}}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} - \frac{m_2}{M_1-\mu}\right)\gamma_5 H_2^0,$$
$$\chi_2 \approx W^3 - \frac{\mathrm{i}}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} + \frac{m_4}{M_2-\mu}\right)\gamma_5 H_1^0 - \frac{\mathrm{i}}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} - \frac{m_4}{M_2-\mu}\right)\gamma_5 H_2^0,$$
$$\chi_3 \approx \frac{\mathrm{i}m_1}{M_1+\mu}\gamma_5 B - \frac{\mathrm{i}m_3}{M_2+\mu} W^3 + \frac{1}{\sqrt{2}} H_1^0 + \frac{1}{\sqrt{2}} H_2^0,$$
$$\chi_4 \approx -\frac{m_2}{M_1-\mu} B + \frac{m_4}{M_2-\mu} W^3 + \frac{\mathrm{i}}{\sqrt{2}}\gamma_5 H_1^0 - \frac{\mathrm{i}}{\sqrt{2}}\gamma_5 H_2^0. \quad (34)$$

Thus, the imaginary part of the transformations contains the factor corresponding to the redefinition $\chi' = \mathrm{i}\gamma_5\chi$ in the minimal diagonalization procedure. It was checked by direct calculation up to the second order that the diagonalization of the real matrix $\mathbf{M}_{\rm I}'$ with redefinition of the field with negative mass gives the same results when the free parameter is equal to zero (see Appendix B).

It is known that in a wide class of SUSY scenarios the values of $M_{1,2}$ and/or $\mu$ are of the order of TeV and higher, so the coefficients in (34) are of the order of $10^{-1}$ or less. Thus, the mixing terms give a contribution to the physical values $\sim 1\%$, so the expressions (34) can be used in practical calculations with a good accuracy.

## 5 Conclusion

It is known that the diagonalization of the neutralino mass form by the orthogonal real matrix leads to the neutralino mass spectrum with one negative mass. This has to be taken into account in calculation rules or by a redefinition of the field with negative mass. An alternative way is the diagonalization by a unitary complex matrix which leads to the mass spectrum with all positive masses. Formally, both the ways are equivalent, but the second one is more convenient, because it does not demand any modification of the standard calculation rules.

In this work, we have considered the connection between the mass sign, the relative parity of the neutralino states and the structure of the boson–neutralino interaction. These features should be considered in the evaluation of the SI and SD contribution to neutralino–nucleon scattering. The suggested approach directly illustrates the existence of one free parameter, generated by the specific symmetry of the $\mu$-term. When this parameter is equal to zero, both approaches give the same results. This was strictly shown in our work for the mass spectrum and states up to the second approximation.

We suggest a simple and convenient way of diagonalization by a unitary matrix with the exponential parametrization. Having used this matrix, we get transparent perturbative formalization of the diagonalization procedure. This method gives simple expressions, illustrating the neutralino states structure and the form of the gauge boson–neutralino interaction. These expressions can be used in most of the SUSY scenarios with the accuracy $\sim 1\%$ or higher.

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## Appendix A

To explicitly show the appearance of the imaginary unit in the neutralino mass matrix and for consistency we give here the minimal part of the SUSY Lagrangian and briefly describe the transformation of the initial SUSY expressions to the ones in terms of four-dimensional fields. All definitions and calculations are in the notation of [31–33]. We consider the electro-weak part of the MSSM Lagrangian

$$L = L_{\rm G} + L_{\rm H} + L_{\rm Ph}. \quad (A.1)$$

In (A.1), the gauge term $L_{\mathrm{G}}$ has the standard form

$$L_{\mathrm{G}} = \frac{1}{4}\{(W^{\alpha}W_{\alpha})_{\theta\theta} + (\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha})_{\bar\theta\bar\theta} + (W_b^{\alpha}W_{\alpha}^b)_{\theta\theta} + (\bar{W}_{\dot\alpha}^b\bar{W}_b^{\dot\alpha})_{\bar\theta\bar\theta}\}, \quad (A.2)$$

where $W_\alpha$ and $W_{\dot\alpha}$ are U(1) gauge superfields and $W_\alpha^b$ and $W_{\dot\alpha}^b$ are SU(2) gauge superfields. The Higgs term contains two chiral superfields with hypercharges $Y_{1,2} = \pm 1$,

$$L_{\mathrm{H}} = \{H_1^+ \exp(g_1G_1 - g_2G_2)H_1 + H_2^+ \exp(-g_1G_1 - g_2G_2)H_2\}_{\theta\theta\bar\theta\bar\theta}. \quad (A.3)$$

The phenomenological part contains the so-called $\mu$-term and gauge soft mass terms:

$$L_{\mathrm{Ph}} = \mu[(H_1\epsilon H_2)_{\theta\theta} + (H_1^+\epsilon H_2^+)_{\bar\theta\bar\theta}] - \frac{1}{2}M_1(bb + \bar{b}\bar{b}) - \frac{1}{2}M_2(\omega_a\omega_a + \bar\omega_a\bar\omega_a), \quad (A.4)$$

where $\epsilon = \mathrm{i}\tau_2$. To define the notation of the components we also present the expressions for the gauge superfields $G_1$ and $G_2$ in Wess–Zumino gauge:

$$G_1 = \theta\sigma^\mu\bar\theta \cdot B_\mu + \mathrm{i}\theta\theta\cdot\bar\theta\bar{b} - \mathrm{i}\bar\theta\bar\theta\cdot\theta b + \frac{1}{2}\theta\theta\cdot\bar\theta\bar\theta\cdot D_1,$$
$$G_2^a = \theta\sigma^\mu\bar\theta\cdot W_\mu^a + \mathrm{i}\theta\theta\cdot\bar\theta\bar\omega^a - \mathrm{i}\bar\theta\bar\theta\cdot\theta\omega^a + \frac{1}{2}\theta\theta\cdot\bar\theta\bar\theta\cdot D_2^a,$$
$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}D_\alpha G, \qquad \bar{W}_{\dot\alpha} = -\frac{1}{4}DD\bar{D}_{\dot\alpha}G. \quad (A.5)$$

The Higgs chiral superfields are

$$H_1 = h_u + \sqrt{2}\theta h_1 + \theta\theta\cdot F_1, \quad H_2 = h_d + \sqrt{2}\theta h_2 + \theta\theta\cdot F_2. \quad (A.6)$$

Thus, the particle content is

$$G_1 = (B_\mu, b), \qquad G_2^a = (W_\mu^a, \omega^a),$$
$$H_1 = (h_u, h_1), \qquad H_2 = (h_d, h_2), \quad (A.7)$$

where

$$h_u = \begin{pmatrix} h_u^0 \\ h_u^- \end{pmatrix}, \quad h_1 = \begin{pmatrix} h_1^0 \\ h_1^- \end{pmatrix}, \quad h_d = \begin{pmatrix} h_d^+ \\ h_d^0 \end{pmatrix}, \quad h_2 = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix}. \quad (A.8)$$

In (A.7) and (A.8) $B_\mu, W_\mu^a, h_u, h_d$ are boson fields and $b, \omega^a, h_1, h_2$ are two-component fermions.

Using the standard method from (A.1)–(A.4) we get Lagrangians in terms of two-component fermions. The gauge field Lagrangian is

$$L_{\mathrm{G}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \mathrm{i}\bar{b}\bar\sigma^\mu\partial_\mu b + \frac{1}{2}D_1^2 - \frac{1}{4}W_a^{\mu\nu}W_{\mu\nu}^a + \mathrm{i}\omega^a\sigma^\mu(\partial_\mu\bar\omega_a + g_2\epsilon_{abc}\bar\omega_c W_\mu^b) + \frac{1}{2}D_2^a D_{2a}, \quad (A.9)$$

where $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2\epsilon^{abc}W_{\mu b}W_{\nu c}$. The Higgs field Lagrangian is

$$\begin{aligned} L_{\mathrm{H}} = &\mathrm{i}\bar{h}_1\bar\sigma^\mu\partial_\mu h_1 + \mathrm{i}\bar{h}_2\bar\sigma^\mu\partial_\mu h_2 \\ &- \frac{1}{2}g_1B_\mu\bar{h}_1\bar\sigma^\mu h_1 + \frac{1}{2}g_1B_\mu\bar{h}_2\bar\sigma^\mu h_2 \\ &+ \frac{1}{2}g_2W_\mu^a\bar{h}_1\bar\sigma^\mu\tau_a h_1 + \frac{1}{2}g_2W_\mu^a\bar{h}_2\bar\sigma^\mu\tau_a h_2 \\ &+ \frac{\mathrm{i}g_1}{\sqrt2}(h_u^+bh_1 - \bar{h}_1\bar{b}h_u) - \frac{\mathrm{i}g_1}{\sqrt2}(h_d^+bh_2 - \bar{h}_2\bar{b}h_d) \\ &- \frac{\mathrm{i}g_2}{\sqrt2}(h_u^+\omega h_1 - \bar{h}_1\bar\omega h_u) - \frac{\mathrm{i}g_2}{\sqrt2}(h_d^+\omega h_2 - \bar{h}_2\bar\omega h_d). \end{aligned} \quad (A.10)$$

The phenomenological Lagrangian is

$$L_{\mathrm{Ph}} = -\mu(h_1\epsilon h_2 + \bar{h}_1\epsilon\bar{h}_2) - \frac{1}{2}M_1(bb + \bar{b}\bar{b}) - \frac{1}{2}M_2(\omega_a\omega_2 + \bar\omega\bar\omega). \quad (A.11)$$

In (A.9)–(A.11) all fermion fields are two-component spinors. The transition to four-component Mayorana spinors in a Weyl basis is defined by the following relations:

$$\chi_k = \begin{pmatrix} \phi_k \\ \bar\phi_k \end{pmatrix}, \quad \phi_k = (b, \omega^a, h_1, h_2);$$
$$\chi_{kL} = \begin{pmatrix} \phi_k \\ 0 \end{pmatrix}, \quad \chi_{kR} = \begin{pmatrix} 0 \\ \bar\phi_k \end{pmatrix}, \quad \chi_{\mathrm{L}}^{\mathrm{C}} = \chi_{\mathrm{R}} = R\chi;$$
$$L = \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad R = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \quad (A.12)$$

With (A.12) we get

$$\mathrm{i}\bar{b}\bar\sigma^\mu\partial_\mu b = \frac{\mathrm{i}}{2}\bar{B}\gamma^\mu\partial_\mu B,$$
$$bb + \bar{b}\bar{b} = \bar{B}B, \quad B = \begin{pmatrix} b \\ \bar{b} \end{pmatrix};$$
$$\mathrm{i}\omega^a\sigma^\mu\partial_\mu\bar\omega_a = \frac{1}{2}\bar{W}^a\gamma^\mu\partial_\mu W^a + \mathrm{Div}(W),$$
$$\omega_a\omega^a + \bar\omega_a\bar\omega^a = \bar{W}_aW^a, \quad W^a = \begin{pmatrix} \omega^a \\ \bar\omega^a \end{pmatrix};$$
$$\mathrm{i}g_2\omega^a\sigma^\mu\epsilon_{abc}\bar\omega_cW_\mu^b = \frac{\mathrm{i}}{2}g_2\bar{W}^a\gamma^\mu W^c\epsilon_{abc}W_\mu^b;$$
$$\mathrm{i}\bar{h}_1\bar\sigma^\mu\partial_\mu h_1 = \frac{\mathrm{i}}{2}\bar{H}_1\gamma^\mu\partial_\mu H_1,$$
$$\bar{h}_1\bar\sigma^\mu h_1 = \bar{H}_1\gamma^\mu H_{1\mathrm{L}}, \quad \bar{h}_2\bar\sigma^\mu h_2 = \bar{H}_2\gamma^\mu H_{2\mathrm{L}};$$
$$\bar{h}_1\bar\sigma^\mu\tau_a h_1 = \bar{H}_1\gamma^\mu\tau_a H_{1\mathrm{L}},$$
$$\bar{h}_2\bar\sigma^\mu\tau_a h_2 = \bar{H}_2\gamma^\mu\tau_a H_{2\mathrm{L}}, \quad H_a = \begin{pmatrix} h^a \\ \bar{h}^a \end{pmatrix};$$
$$h_u^+bh_1 - \bar{h}_1\bar{b}h_u = h_u^+\bar{B}H_{1\mathrm{L}} - \bar{H}_{1\mathrm{L}}Bh_u,$$
$$h_d^+bh_2 - \bar{h}_2\bar{b}h_d = h_d^+\bar{B}H_{2\mathrm{L}} - \bar{H}_{2\mathrm{L}}Bh_d;$$
$$h_u^+\omega h_1 - \bar{h}_1\bar\omega h_u = h_u^+\bar{W}H_{1\mathrm{L}} - \bar{H}_{1\mathrm{L}}Wh_u,$$
$$h_d^+\omega h_2 - \bar{h}_2\bar\omega h_d = h_d^+\bar{W}H_{2\mathrm{L}} - \bar{H}_{2\mathrm{L}}Wh_d;$$
$$h_1\epsilon h_2 + \bar{h}_1\epsilon\bar{h}_2 = \bar{H}_{1R}\epsilon H_{2\mathrm{L}} + \bar{H}_{1\mathrm{L}}\epsilon H_{2\mathrm{R}}. \quad (A.13)$$

From (A.9)–(A.11) with the help of (A.12) and (A.13) we obtain Lagrangians in terms of four-component spinors. The gauge field Lagrangian is

$$L_{\mathrm{G}} = -\frac{1}{4}B^{\mu\nu}B_{\mu\nu} + \frac{\mathrm{i}}{2}\bar{B}\gamma^{\mu}\partial_{\mu}B + \frac{1}{2}D_1^2 - \frac{1}{4}W_a^{\mu\nu}W_{\mu\nu}^a + \frac{\mathrm{i}}{2}\bar{W}^a\gamma^{\mu}\left(\partial_{\mu}W_a + g_2\epsilon_{abc}W^cW_{\mu}^b\right) + \frac{1}{2}D_2^aD_{2a}\,. \tag{A.14}$$

The Higgs fermion field Lagrangian is

$$\begin{aligned} L_{\mathrm{H}} = &\frac{\mathrm{i}}{2}\bar{H}_1\gamma^{\mu}\left(\partial_{\mu}H_1 + \mathrm{i}g_1B_{\mu}H_{1\mathrm{L}} - \mathrm{i}g_2W_{\mu}^a\tau_aH_{1\mathrm{L}}\right)\\ &+\frac{\mathrm{i}}{2}\bar{H}_2\gamma^{\mu}\left(\partial_{\mu}H_2 - \mathrm{i}g_1B_{\mu}H_{2\mathrm{L}} - \mathrm{i}g_2W_{\mu}^a\tau_aH_{2\mathrm{L}}\right)\\ &+\frac{\mathrm{i}g_1}{\sqrt{2}}(h_u^+\bar{B}H_{1\mathrm{L}} - \bar{H}_{1\mathrm{L}}Bh_u)\\ &-\frac{\mathrm{i}g_1}{\sqrt{2}}(h_d^+\bar{B}H_{2\mathrm{L}} - \bar{H}_{2\mathrm{L}}Bh_d)\\ &-\frac{\mathrm{i}g_2}{\sqrt{2}}(h_u^+\bar{W}H_{1\mathrm{L}} - \bar{H}_{1\mathrm{L}}Wh_u)\\ &-\frac{\mathrm{i}g_2}{\sqrt{2}}(h_d^+\bar{W}H_{2\mathrm{L}} - \bar{H}_{2\mathrm{L}}Wh_d)\,. \end{aligned} \tag{A.15}$$

The phenomenological Lagrangian is

$$L_{\mathrm{Ph}} = -\frac{1}{2}M_1\bar{B}B - \frac{1}{2}M_2\bar{W}_aW^a - \mu(\bar{H}_{1R}\epsilon H_{2\mathrm{L}} + \bar{H}_{1\mathrm{L}}\epsilon H_{2\mathrm{R}}). \tag{A.16}$$

## Appendix B

Here we consider a simple and easily formalized method of the complex mass form diagonalization:

$$\left(\bar{\phi}_{\mathrm{R}}^{\mathrm{I}}\right)^{\mathrm{T}}\mathbf{M}_{\mathrm{I}}\phi_{\mathrm{L}}^{\mathrm{I}} + \mathrm{h.c.} = (\bar{\chi}_{\mathrm{R}})^{\mathrm{T}}\mathbf{U}_{\mathrm{I}}^{\mathrm{T}}\mathbf{M}_{\mathrm{I}}\mathbf{U}_{\mathrm{I}}\chi_{\mathrm{L}} + \mathrm{h.c.} = m_i\bar{\chi}_i\chi_i\,. \tag{B.1}$$

In (B.1) $\mathbf{M}_{\mathrm{I}}$ is a symmetric complex matrix, $\phi_{\mathrm{R,L}}^{\mathrm{I}}$ are the chiral components of the initial Majorana spinor fields arising after intermediate diagonalization (20), and $\chi$ are the final Majorana fields (neutralino):

$$(\phi^{\mathrm{I}})^{\mathrm{T}} = \left(B, W^3, \phi_3^{\mathrm{I}}, \phi_4^{\mathrm{I}}\right), \quad \chi^{\mathrm{T}} = (\chi_1, \chi_2, \chi_3, \chi_4)\,. \tag{B.2}$$

The intermediate states $\phi_3^{\mathrm{I}}$ and $\phi_4^{\mathrm{I}}$ are defined by (7). We suggest straightforward diagonalization of the form (B.1) by the unitary matrix in the exponential parametrization. In the general case, the unitary matrix $\mathbf{U}(n\times n)$ has $2n^2 - n^2 = n^2$ parameters, where $n^2$ unitary conditions are taken into account. For $n = 4$ we have 16 input parameters, six angles and ten phases [12]. However, in the case of a symmetric mass matrix with real $M_{1,2}$ and $\mu$, this number of parameters is excessive, so we suggest a unitary matrix with six angle and six phase input parameters. These 12 parameters can be defined from 12 independent conditions following from the symmetric matrix diagonalization $(\mathbf{U}_{\mathrm{I}}^{\mathrm{T}}\mathbf{M}_{\mathrm{I}}\mathbf{U}_{\mathrm{I}})_{ik} = 0$, $i > k$ (or $i < k$). It is convenient for the analysis to use $\mathbf{U}_{\mathrm{I}}$ in the exponential form [34, 35]:

$$\mathbf{U}_{\mathrm{I}} = \begin{pmatrix} a_1 & \delta_1\mathrm{e}^{-\mathrm{i}\phi_1} & \delta_2\mathrm{e}^{-\mathrm{i}\phi_2} & \delta_3\mathrm{e}^{-\mathrm{i}\phi_3}\\ r_1\mathrm{e}^{\mathrm{i}\alpha_1} & a_2 & \delta_4\mathrm{e}^{-\mathrm{i}\phi_4} & \delta_5\mathrm{e}^{-\mathrm{i}\phi_5}\\ r_2\mathrm{e}^{\mathrm{i}\alpha_2} & r_4\mathrm{e}^{\mathrm{i}\alpha_4} & a_3 & \delta_6\mathrm{e}^{-\mathrm{i}\phi_6}\\ r_3\mathrm{e}^{\mathrm{i}\alpha_3} & r_5\mathrm{e}^{\mathrm{i}\alpha_5} & r_6\mathrm{e}^{\mathrm{i}\alpha_6} & a_4 \end{pmatrix}. \tag{B.3}$$

In (B.3) $\delta_1$–$\delta_6$ and $\phi_1$–$\phi_6$ are the angle and phase parameters, respectively. The quantities $\delta_k$ and $\phi_k$ are the input parameters, while $a_\beta$ and $r_k, \alpha_k$ are some functions of the input parameters which follow from the unitary condition $\mathbf{U}^+\mathbf{U} = \mathbf{1}$:

$$\begin{aligned} &a_1 = \left(1 - \delta_1^2 - \delta_2^2 - \delta_3^2\right)^{1/2}, \quad a_2 = \left(1 - \delta_4^2 - \delta_5^2 - r_1^2\right)^{1/2},\\ &a_3 = \left(1 - \delta_6^2 - r_2^2 - r_4^2\right)^{1/2}, \quad a_4 = \left(1 - r_3^2 - r_5^2 - r_6^2\right)^{1/2},\\ &a_1r_1\mathrm{e}^{\mathrm{i}\alpha_1} + a_2\delta_1\mathrm{e}^{\mathrm{i}\phi_1} + \delta_2\delta_4\mathrm{e}^{\mathrm{i}(\phi_2-\phi_4)} + \delta_3\delta_5\mathrm{e}^{\mathrm{i}(\phi_3-\phi_5)} = 0\,,\\ &a_1r_2\mathrm{e}^{\mathrm{i}\alpha_2} + \delta_1r_4\mathrm{e}^{\mathrm{i}(\alpha_4+\phi_1)} + a_3\delta_2\mathrm{e}^{\mathrm{i}\phi_2} + \delta_3\delta_6\mathrm{e}^{\mathrm{i}(\phi_3-\phi_6)} = 0\,,\\ &a_1r_3\mathrm{e}^{\mathrm{i}\alpha_3} + \delta_1r_5\mathrm{e}^{\mathrm{i}(\alpha_5+\phi_1)} + \delta_2r_6\mathrm{e}^{\mathrm{i}(\phi_2+\alpha_6)} + a_4\delta_3\mathrm{e}^{\mathrm{i}\phi_3} = 0\,,\\ &r_1r_3\mathrm{e}^{\mathrm{i}(\alpha_3-\alpha_1)} + a_2r_5\mathrm{e}^{\mathrm{i}\alpha_5} + \delta_4r_6\mathrm{e}^{\mathrm{i}(\phi_4+\alpha_6)} + a_4\delta_5\mathrm{e}^{\mathrm{i}\phi_5} = 0\,,\\ &r_1r_2\mathrm{e}^{\mathrm{i}(\alpha_2-\alpha_1)} + a_2r_4\mathrm{e}^{\mathrm{i}\alpha_4} + a_3\delta_4\mathrm{e}^{\mathrm{i}\phi_4} + \delta_5\delta_6\mathrm{e}^{\mathrm{i}(\phi_5-\phi_6)} = 0\,,\\ &r_2r_3\mathrm{e}^{\mathrm{i}(\alpha_3-\alpha_2)} + r_4r_5\mathrm{e}^{\mathrm{i}(\alpha_5-\alpha_4)} + a_3r_6\mathrm{e}^{\mathrm{i}\alpha_6} + a_4\delta_6\mathrm{e}^{\mathrm{i}\phi_6} = 0\,. \end{aligned} \tag{B.4}$$

The Ansatz (B.3) is convenient for approximate calculations in the case of a quasidiagonal mass matrix, for instance, $\mathbf{M}_{\mathrm{I}}$ defined by (21). In the case considered, the absolute values of the diagonal elements and the differences are much greater than the off-diagonal ones (the equality of the third and fourth diagonal elements $\mu$ in $\mathbf{M}_{\mathrm{I}}$ is compensated by off-diagonal zero). The diagonalyzing matrix has a similar structure, i.e. the input parameters $\delta_k$ in (B.4) are small, $\delta_k \ll 1$ and $a_k \simeq 1$. So, due to the smallness of parameters $\delta_k \sim M_Z/M_a$, where $a = 1, 2$, one can easily solve the system of equations (B.4) approximately. The functions $a_\alpha, r_k$ and $\alpha_k$ are determined from (B.4), the input parameters $\delta_k$ and $\phi_k$ are defined by the conditions $(\mathbf{U}_{\mathrm{I}}^{\mathrm{T}}\mathbf{M}_{\mathrm{I}}\mathbf{U}_{\mathrm{I}})_{ik} = 0$, $i > k$. Hence, the diagonal elements $(\mathbf{U}_{\mathrm{I}}^{\mathrm{T}}\mathbf{M}_{\mathrm{I}}\mathbf{U}_{\mathrm{I}})_{kk} = m_k$ give the masses in terms of known quantities.

Finally, we have done the calculations up to the second order $\sim M_Z^2/M_a^2$ (or $M_Z^2/\mu^2$) inclusively and get the expressions for the elements of the diagonalizing matrix (B.3) (the hierarchy of $M_a$ and $\mu$ is arbitrary). Such an approximation is reasonable for calculations within a wide class of split SUSY models.

The input parameters are

$$\begin{aligned} &\delta_1\mathrm{e}^{-\mathrm{i}\phi_1} = \frac{M_Z^2\sin 2\theta_{\mathrm{W}}}{2(M_2^2-\mu^2)}\frac{M_2+\mu\sin 2\beta}{M_1-M_2}\,,\\ &\delta_2\mathrm{e}^{-\mathrm{i}\phi_2} = \mathrm{i}\frac{m_1}{M_1+\mu}\,, \qquad \delta_3\mathrm{e}^{-\mathrm{i}\phi_3} = -\frac{m_2}{M_1-\mu}\,,\\ &\delta_4\mathrm{e}^{-\mathrm{i}\phi_4} = -\mathrm{i}\frac{m_3}{M_2+\mu}\,, \qquad \delta_5\mathrm{e}^{-\mathrm{i}\phi_5} = \frac{m_4}{M_2-\mu}\,,\\ &\delta_6\mathrm{e}^{-\mathrm{i}\phi_6} = -\frac{\mathrm{i}}{2\mu}\left(\frac{m_1m_2}{M_1-\mu} + \frac{m_3m_4}{M_2-\mu}\right). \end{aligned} \tag{B.5}$$

The diagonal elements $a_\beta$ are

$$a_1 = 1 - \frac{1}{2}\left[\frac{m_1^2}{(M_1+\mu)^2} + \frac{m_2^2}{(M_1-\mu)^2}\right],$$
$$a_2 = 1 - \frac{1}{2}\left[\frac{m_3^2}{(M_2+\mu)^2} + \frac{m_4^2}{(M_2-\mu)^2}\right],$$
$$a_3 = 1 - \frac{1}{2}\left[\frac{m_1^2}{(M_1+\mu)^2} + \frac{m_3^2}{(M_2+\mu)^2}\right],$$
$$a_4 = 1 - \frac{1}{2}\left[\frac{m_2^2}{(M_1-\mu)^2} + \frac{m_4^2}{(M_2-\mu)^2}\right]. \tag{B.6}$$

The off-diagonal elements are

$$r_1 e^{i\alpha_1} = \frac{m_1 m_3}{(M_1+\mu)(M_2+\mu)} + \frac{m_2 m_4}{(M_1-\mu)(M_2-\mu)} - \frac{M_Z^2 \sin 2\theta_W (M_2 + \mu \sin 2\beta)}{2\left(M_2^2-\mu^2\right)(M_1-M_2)},$$
$$r_2 e^{i\alpha_2} = \frac{i m_1}{M_1+\mu}, \qquad r_3 e^{i\alpha_3} = \frac{m_2}{M_1-\mu},$$
$$r_4 e^{i\alpha_4} = -\frac{i m_3}{M_2+\mu}, \qquad r_5 e^{i\alpha_5} = -\frac{m_4}{M_2-\mu},$$
$$r_6 e^{i\alpha_6} = \frac{m_1 m_2}{M_1^2-\mu^2} + \frac{m_3 m_4}{M_2^2-\mu^2} - \frac{i}{2\mu}\left(\frac{m_1 m_2}{M_1-\mu} + \frac{m_3 m_4}{M_2-\mu}\right). \tag{B.7}$$

In (B.5)–(B.7) we give the zero value to the free parameter $\delta_6^0$ arising in the first order. It was checked in the first approximation that the existence of the free parameter $\delta_6^0$ leads to the phase redefinition of the fields $\chi_3$ and $\chi_4$. Having applied (B.5)–(B.7), we obtain expressions for the neutralino masses (31):

$$\left(\mathbf{U}_I^T \mathbf{M}_I \mathbf{U}_I\right)_{kk} = m_k, \tag{B.8}$$

which are positive. We have checked also the unitary condition $\mathbf{U}_I^+ \mathbf{U}_I = \mathbf{1}$.

The structure of the neutralino chiral fields results from the transformations

$$\phi_L = \mathbf{U}\chi_L, \qquad \phi_R = (\phi_L)^C = \mathbf{U}^*\chi_R,$$
$$\chi_L = \mathbf{U}^{-1}\phi_L = \mathbf{U}^+\phi_L, \qquad \chi_R = (\chi_L)^C = \mathbf{U}^T\phi_R, \tag{B.9}$$

where $\mathbf{U} = \mathbf{U}_2 \cdot \mathbf{U}_I$, $\mathbf{U}_2$ is defined by (4) and $(\phi)^T = (B, W^3, H_1^0, H_2^0)$.

To illustrate the relation between the initial and physical fields we give transformations in the first order of mixing:

$$B_L \approx \chi_{1L} + \frac{i m_1}{M_1+\mu}\chi_{3L} - \frac{m_2}{M_1-\mu}\chi_{4L},$$
$$W_L^3 \approx \chi_{2L} - \frac{i m_3}{M_2+\mu}\chi_{3L} + \frac{m_4}{M_2-\mu}\chi_{4L},$$
$$H_{1L}^0 \approx \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} + \frac{m_2}{M_1-\mu}\right)\chi_{1L} - \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} + \frac{m_4}{M_2-\mu}\right)\chi_{2L} + \frac{1}{\sqrt{2}}\chi_{3L} + \frac{i}{\sqrt{2}}\chi_{4L},$$
$$H_{2L}^0 \approx \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} - \frac{m_2}{M_1-\mu}\right)\chi_{1L} - \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} - \frac{m_4}{M_2-\mu}\right)\chi_{2L} + \frac{1}{\sqrt{2}}\chi_{3L} - \frac{i}{\sqrt{2}}\chi_{4L}. \tag{B.10}$$

Transformation of the $R$-component can easily be found from the relation $\chi_R = (\chi_L)^C$. Inverse transformations illustrate the neutralino structure:

$$\chi_{1L} \approx B_L - \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} + \frac{m_2}{M_1-\mu}\right)H_{1L}^0 - \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} - \frac{m_2}{M_1-\mu}\right)H_{2L}^0,$$
$$\chi_{2L} \approx W_L^3 + \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} + \frac{m_4}{M_2-\mu}\right)H_{1L}^0 + \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} - \frac{m_4}{M_2-\mu}\right)H_{2L}^0,$$
$$\chi_{3L} \approx -\frac{i m_1}{M_1+\mu}B_L + \frac{i m_3}{M_2+\mu}W_L^3 + \frac{1}{\sqrt{2}}H_{1L}^0 + \frac{1}{\sqrt{2}}H_{2L}^0,$$
$$\chi_{4L} \approx -\frac{m_2}{M_1-\mu}B_L + \frac{m_4}{M_2-\mu}W_L^3 - \frac{i}{\sqrt{2}}H_{1L}^0 + \frac{i}{\sqrt{2}}H_{2L}^0. \tag{B.11}$$

The transformation of the non-chiral Majorana field $\chi = \chi_L + \chi_R$ is

$$\phi = (\mathrm{Re}\,\mathbf{U} - i\gamma_5\,\mathrm{Im}\,\mathbf{U})\chi, \quad \chi = \left(\mathrm{Re}\,\mathbf{U}^T + i\gamma_5\,\mathrm{Im}\,\mathbf{U}^T\right)\phi. \tag{B.12}$$

In the first order of mixing from (B.12) it follows that

$$B \approx \chi_1 - \frac{i m_1}{M_1+\mu}\gamma_5\chi_3 - \frac{m_2}{M_1-\mu}\chi_4,$$
$$W^3 \approx \chi_2 + \frac{i m_3}{M_2+\mu}\gamma_5\chi_3 + \frac{m_4}{M_2-\mu}\chi_4,$$
$$H_1^0 \approx -\frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} + \frac{m_2}{M_1-\mu}\right)\gamma_5\chi_1 + \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} + \frac{m_4}{M_2-\mu}\right)\gamma_5\chi_2 + \frac{1}{\sqrt{2}}\chi_3 - \frac{i}{\sqrt{2}}\gamma_5\chi_4,$$
$$H_2^0 \approx -\frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} - \frac{m_2}{M_1-\mu}\right)\gamma_5\chi_1 + \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} - \frac{m_4}{M_2-\mu}\right)\gamma_5\chi_2 + \frac{1}{\sqrt{2}}\chi_3 + \frac{i}{\sqrt{2}}\gamma_5\chi_4. \tag{B.13}$$

The structure of non-chiral neutralino states is

$$\chi_1 \approx B + \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} + \frac{m_2}{M_1-\mu}\right)\gamma_5 H_1^0 + \frac{i}{\sqrt{2}}\left(\frac{m_1}{M_1+\mu} - \frac{m_2}{M_1-\mu}\right)\gamma_5 H_2^0,$$
$$\chi_2 \approx W^3 - \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} + \frac{m_4}{M_2-\mu}\right)\gamma_5 H_1^0 - \frac{i}{\sqrt{2}}\left(\frac{m_3}{M_2+\mu} - \frac{m_4}{M_2-\mu}\right)\gamma_5 H_2^0,$$

$$\chi_3 \approx +\frac{\mathrm{i}m_1}{M_1+\mu}\gamma_5 B - \frac{\mathrm{i}m_3}{M_2+\mu}W^3 + \frac{1}{\sqrt{2}}H_1^0 + \frac{1}{\sqrt{2}}H_2^0\,,$$
$$\chi_4 \approx -\frac{m_2}{M_1-\mu}B + \frac{m_4}{M_2-\mu}W^3 + \frac{\mathrm{i}}{\sqrt{2}}\gamma_5 H_1^0 - \frac{\mathrm{i}}{\sqrt{2}}\gamma_5 H_2^0\,. \quad \text{(B.14)}$$

Thus, the imaginary part of the transformations contains the factor corresponding to the redefinition $\chi' = \mathrm{i}\gamma_5\chi$ in the intermediate diagonalization procedure. By direct calculation (up to the second order) it was checked that the diagonalization of the real matrix $\mathbf{M}'_\mathrm{I}$ with redefinition of the final field with negative mass gives the same results as in our case with $\delta_6^0 = 0$. Note that our formulae (B.10)–(B.14) coincide with the corresponding ones from [12] if we redefine the neutralino state with negative mass as $\chi \to \mathrm{i}\gamma_5\chi$.

Now we generalize the method of diagonalization for the case of a mass matrix with complex parameters $M_1\mathrm{e}^{\mathrm{i}\psi_1}$, $M_2\mathrm{e}^{\mathrm{i}\psi_2}$ and $\mu\mathrm{e}^{\mathrm{i}\psi_\mu}$. Then we have to extend (B.3), introducing additional phase parameters according to $a_\beta \to a_\beta \mathrm{e}^{\mathrm{i}\xi_k}$. The functions $a_\beta, r_k$ and $\alpha_k$ are determined in terms of the input parameters $\delta_k, \phi_k$ and $\xi_\beta$ by the unitary condition $\mathbf{U}^+\mathbf{U} = \mathbf{1}$. We represent them in the form

$$a_1 = \left(1-\delta_1^2-\delta_2^2-\delta_3^2\right)^{1/2}, \quad a_2 = \left(1-\delta_1^2-\delta_4^2-r_5^2\right)^{1/2},$$
$$a_3 = \left(1-\delta_2^2-r_4^2-r_6^2\right)^{1/2}, \quad a_4 = \left(1-r_3^2-r_5^2-r_6^2\right)^{1/2},$$
$$a_1 r_1 \mathrm{e}^{\mathrm{i}(\alpha_1+\xi_1)} + a_2\delta_1 \mathrm{e}^{\mathrm{i}(\phi_1-\xi_2)} + \delta_2\delta_4 \mathrm{e}^{\mathrm{i}(\phi_2-\phi_4)} + \delta_3\delta_5 \mathrm{e}^{\mathrm{i}(\phi_3-\phi_5)} = 0\,,$$
$$a_1 r_2 \mathrm{e}^{\mathrm{i}(\alpha_2+\xi_1)} + \delta_1 r_4 \mathrm{e}^{\mathrm{i}(\alpha_4+\phi_1)} + a_3\delta_2 \mathrm{e}^{\mathrm{i}(\phi_2-\xi_3)} + \delta_3\delta_6 \mathrm{e}^{\mathrm{i}(\phi_3-\phi_6)} = 0\,,$$
$$a_1 r_3 \mathrm{e}^{\mathrm{i}(\alpha_3+\xi_1)} + \delta_1 r_5 \mathrm{e}^{\mathrm{i}(\alpha_5+\phi_1)} + \delta_2 r_6 \mathrm{e}^{\mathrm{i}(\phi_2+\alpha_6)} + a_4\delta_3 \mathrm{e}^{\mathrm{i}(\phi_3-\xi_4)} = 0\,,$$
$$r_1 r_2 \mathrm{e}^{\mathrm{i}(\alpha_2-\alpha_1)} + a_2 r_4 \mathrm{e}^{\mathrm{i}(\alpha_4+\xi_2)} + a_3\delta_4 \mathrm{e}^{\mathrm{i}(\phi_4-\xi_3)} + \delta_5\delta_6 \mathrm{e}^{\mathrm{i}(\phi_5-\phi_6)} = 0\,,$$
$$r_1 r_3 \mathrm{e}^{\mathrm{i}(\alpha_3-\alpha_1)} + a_2 r_5 \mathrm{e}^{\mathrm{i}(\alpha_5+\xi_2)} + \delta_4 r_6 \mathrm{e}^{\mathrm{i}(\phi_4+\alpha_6)} + a_4\delta_5 \mathrm{e}^{\mathrm{i}(\phi_5-\xi_4)} = 0\,,$$
$$r_2 r_3 \mathrm{e}^{\mathrm{i}(\alpha_3-\alpha_2)} + r_4 r_5 \mathrm{e}^{\mathrm{i}(\alpha_5-\alpha_4)} + a_3 r_6 \mathrm{e}^{\mathrm{i}(\alpha_6+\xi_2)} + a_4\delta_6 \mathrm{e}^{\mathrm{i}(\phi_6-\xi_4)} = 0\,. \quad \text{(B.15)}$$

The set of input parameters $\delta_k, \phi_k$ and $\xi_\beta$ is determined in terms of the mass matrix elements utilizing the diagonalization conditions $(\mathbf{U}^\mathrm{T}\mathbf{M}\mathbf{U})_{ik} = 0$, $i \neq k$ and $\mathrm{Im}(\mathbf{U}^\mathrm{T}\mathbf{M}\mathbf{U})_{ii} = 0$.

In the first approximation from the second condition we get $\xi_1 = \psi_1/2, \xi_2 = \psi_2/2$ and $\xi_3 = \xi_4 = \psi_\mu/2$. From the first condition in the same approximation we get

$$\delta_1 = 0 \quad \text{or} \quad \phi_1 = \psi_1/2\,, \quad M_1 = M_2\,;$$
$$\delta_2 = \frac{-m_1}{M_1^2-\mu^2}\left[M_1^2+\mu^2-2\mu M_1\cos(\psi_1+\psi_\mu)\right]^{1/2},$$
$$\tan\left(\phi_2 - \frac{\psi_1}{2}\right) = -\frac{M_1-\mu}{M_1+\mu}\cot\left(\frac{\psi_1+\psi_\mu}{2}\right);$$
$$\delta_3 = \frac{-m_2}{M_1^2-\mu^2}\left[M_1^2+\mu^2+2\mu M_1\cos(\psi_1+\psi_\mu)\right]^{1/2},$$
$$\tan\left(\phi_3 - \frac{\psi_1}{2}\right) = \frac{M_1-\mu}{M_1+\mu}\tan\left(\frac{\psi_1+\psi_\mu}{2}\right);$$
$$\delta_4 = \frac{m_3}{M_2^2-\mu^2}\left[M_2^2+\mu^2-2\mu M_2\cos(\psi_2+\psi_\mu)\right]^{1/2},$$
$$\tan\left(\phi_4 - \frac{\psi_2}{2}\right) = -\frac{M_2-\mu}{M_2+\mu}\cot\left(\frac{\psi_2+\psi_\mu}{2}\right);$$
$$\delta_5 = \frac{m_4}{M_2^2-\mu^2}\left[M_2^2+\mu^2+2\mu M_2\cos(\psi_2+\psi_\mu)\right]^{1/2},$$
$$\tan\left(\phi_5 - \frac{\psi_2}{2}\right) = \frac{M_2-\mu}{M_2+\mu}\tan\left(\frac{\psi_2+\psi_\mu}{2}\right);$$
$$\delta_6 = 0 \quad \text{or} \quad \phi_6 = (\psi_2+\psi_\mu)/4\,. \quad \text{(B.16)}$$

All expressions for the input phases can be represented as explicit functions, for example

$$\phi_2 = \frac{\psi_1}{2} - \arctan\left(\frac{M_1-\mu}{M_1+\mu}\cot\left(\frac{\psi_1+\psi_\mu}{2}\right)\right). \quad \text{(B.17)}$$

From (B.16) one can see that taking account of the complex degrees of freedom in the mass matrix complicates the calculations. However, the perturbation scheme of the method is retained and can easily be formalized. It has to be noted also that we use the same number of phases as in the general method [12], and the question of optimization of this number for a special kind of mass matrix is practically important.

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